1. The strain components $\varepsilon_x = 520 \ \mu e$, $\varepsilon_y = -650 \ \mu e$, and $\gamma_{xy} = 750 \ \mu rad$ are given for a point in a body subjected to plane strain.

a. (4 points) Sketch the deformed shape on the element to the right.

\[ \theta = 35^\circ \]

b. (9 points) Determine the strain components $\varepsilon_x$, $\varepsilon_y$, and $\gamma_{xy}$ at the point if the $x$-axes are rotated with respect to the $xy$-axes by $\theta = 35^\circ$, as illustrated in the figure. Do not sketch the deformed shape of the element.

\[ \varepsilon_x = \frac{487.5}{2} \ \mu e \]
\[ \varepsilon_y = -\frac{617.5}{2} \ \mu e \]
\[ \gamma_{xy} = \frac{842.9}{2} \ \mu rad \]

\[ \text{c. (14 points) Determine the angles } \phi_x \text{ and } \phi_y \text{, principal strains, and maximum in-plane shear strain at the point. Do not sketch the deformed shapes of the element.} \]
\[ \phi_x = \frac{16.33}{2} \text{ deg} = \tan^{-1} \left( \frac{-750}{520+650} \right) \]
\[ \phi_y = -28.67 \ \text{deg} = \phi_x \pm 45^\circ \]
\[ \varepsilon_1 = \frac{629.9}{2} \ \mu e \]
\[ \varepsilon_2 = -\frac{759.9}{2} \ \mu e \]
\[ \gamma_{\text{max}} = \frac{1390}{2} \ \mu rad \]
2. (10 points) The strain rosette shown in the figure was used to obtain normal strain data at a point on the free surface of a machine part. \( \varepsilon_x = 550 \mu \varepsilon \), \( \varepsilon_y = -730 \mu \varepsilon \), and \( \gamma_{xy} = 375 \mu \text{rad} \). Determine the strain components \( \varepsilon_x \), \( \varepsilon_y \), and \( \gamma_{xy} \) at the point.

\[
\varepsilon_x = -730 \mu \varepsilon \\
\varepsilon_y = 550 \mu \varepsilon \\
\gamma_{xy} = 375 \mu \text{rad}
\]

\[
\gamma_{xy} = -\frac{730 + 550}{2} - \frac{730 - 550}{2} \cos 90^\circ + \frac{\gamma_{xy}}{2} \sin -90^\circ
\]

3. (8 points) The strain components \( \varepsilon_x = 290 \mu \varepsilon \), \( \varepsilon_y = 820 \mu \varepsilon \), and \( \gamma_{xy} = -580 \mu \varepsilon \) are given for a point on the free surface of a machine component. The modulus of elasticity for the material is \( E = 73 \text{ GPa} \) and the Poisson’s ratio is \( \nu = 0.30 \). Determine the stresses \( \sigma_x \) and \( \tau_{xy} \) at the point.

\[
\sigma_x = \frac{51.02}{\nu} \text{ MPa} = \frac{73 \times 10^9}{1 - 0.3} \left(390 \times 10^{-6} + 0.3 (820 \times 10^{-6})\right)
\]

\[
\sigma_y = -15.72 \text{ MPa} = \frac{73 \times 10^9}{2(0.3)} (-580 \times 10^{-6})
\]
4. (5 points) A spherical gas-storage tank with an inside diameter of 12 m is being constructed to store gas under an internal pressure of 1.75 MPa. The tank will be constructed from structural steel that has a yield strength of 250 MPa. If a factor of safety of 3.0 with respect to the yield strength is required, determine the minimum wall thickness required for the spherical tank.

\[ t_{\text{min}} = \frac{63 \text{ mm}}{0.5} = \frac{1.75 \times 6 (6)}{2t} \leq \frac{250 \times 6}{3} \]

5. (5 points) A cylindrical boiler with an outside diameter of 3.60 m and a wall thickness of 40 mm is made of a steel alloy that has a yield stress of 415 MPa. Determine the maximum normal stress produced by an internal pressure of 2 MPa.

\[ \sigma_{\text{max}} = 88 \text{ MPa} = \frac{2 \times 6 (1.76)}{0.44} \]

6. A closed cylindrical vessel contains a fluid at a pressure of 620 psi. Assume \( \sigma_{\text{knee}} = 22.32 \text{ kai} \) and \( \sigma_{\text{cyl}} = 11.16 \text{ kai} \). Determine:

   a. (4 points) the absolute maximum shear stress on the outer surface of the cylinder

\[ \tau_{\text{she max}} = 11.6 \text{ kai} \]

   b. (4 points) the absolute maximum shear stress on the inner surface of the cylinder

\[ \tau_{\text{she max}} = 11.52 \text{ kai} \]
7. (9 points) Three loads are applied to the short rectangular post. The cross-sectional dimensions of the post are shown.

a. Using the positive sign convention shown, determine the following internal forces and moments acting on a cross-plane through points $H$ and $K$:

- $F_x = -65$ kN
- $F_y = -210$ kN
- $F_z = -95$ kN
- $M_x = -24.75$ kN-m
- $M_y = 0$ kN-m
- $M_z = 9.75$ kN-m

b. (14 points) For the following loadings, determine the normal and shear stresses at point $H$:

- $F_x = 48$ kN
- $F_y = 0$ kN
- $F_z = 73$ kN
- $M_x = 3$ kN-m
- $M_y = 0$ kN-m
- $M_z = -2.5$ kN-m

\[ \sigma_x = \boxed{0} \text{ MPa} \]
\[ \sigma_y = \boxed{-58.59} \text{ MPa} = \frac{-3000 (0.8)}{12(0.1)^2/12} \]
\[ \tau_{xy} = \boxed{3.75} \text{ MPa} = \frac{48000 (0.03)(0.06)(0.16)}{12(0.1)^3/12} \]
9. (14 points) A solid steel crank has an outside diameter of 30 mm. For the following loadings, determine the normal and shear stresses on the top surface of the crank at point $A$.

\[ F_x = 2350 \text{ N} \]
\[ F_y = -1275 \text{ N} \]
\[ F_z = 0 \text{ N} \]
\[ M_x = 204 \text{ Nm} \]
\[ M_y = 376 \text{ Nm} \]
\[ M_z = 0 \text{ Nm} \]

\[ \sigma_x = \frac{3.325}{\frac{2350}{4}(0.03)^2} \text{ MPa} \]
\[ \sigma_y = 0 \text{ MPa} \]
\[ \tau_{yz} = \frac{-38.48}{\frac{204(0.05)}{32}(0.03)^4} \text{ MPa} \]